

Unit 5 Study Guide

change signs into EQ.

Name: AK S: _____

Use the information provided to write the standard form equation of each circle.

1) Center: $(-5, -7)$
Radius: 9

$$(x+5)^2 + (y+7)^2 = 81$$

2) Center: $(16, -14)$
Radius: $(\sqrt{3})^2 = 3$

$$(x-16)^2 + (y+14)^2 = 3$$

pay attention

Use the information provided to write the general conic form equation of each circle.

3) $(x+10)^2 + (y-7)^2 = 9$ $100 + 49 - 9 = 140$

$$x^2 + 2(x)(10) + 100 + y^2 + 2(y)(-7) + 49 - 9 = 0$$

$$x^2 + y^2 + 20x - 14y + 140 = 0$$

4) $(x-14)^2 + (y+14)^2 = 9$ $x^2 - 28x + 196 + y^2 + 28y + 196 - 9 = 0$

$$x^2 + y^2 - 28x + 28y + 383 = 0$$

Use the information provided to write the standard form equation of each circle. Then, state the center and radius.

5) $x^2 + y^2 - 20x + 2y + 76 = 0$

$\frac{-20}{2} = -10 \Rightarrow (-10)^2 = 100$ $\frac{2}{2} = 1 \Rightarrow 1^2 = 1$

BALANCE

$$x^2 - 20x + 100 + y^2 + 2y + 1 = -76 + 100 + 1$$

$$(x-10)^2 + (y+1)^2 = 25 \quad C: (10, -1) \quad r = 5$$

6) $x^2 + y^2 + 28x + 24y + 331 = 0$

$\frac{28}{2} = 14 \Rightarrow 14^2 = 196$ $\frac{24}{2} = 12 \Rightarrow 12^2 = 144$

BALANCE

$$x^2 + 28x + 196 + y^2 + 24y + 144 = -331 + 196 + 144$$

$$(x+14)^2 + (y+12)^2 = 9 \quad C: (-14, -12) \quad r = 3$$

Use the information provided to write the standard form equation of the circle.

7) Center $(0, -1)$

Point on the Circle $(-13, -10)$

$(0, -1)$ $(-1, -10)$

$\Delta x = (-13) \quad \Delta y = (9)$

dist/r = $\sqrt{(-13)^2 + (9)^2}$

$r = \sqrt{169 + 81} = \sqrt{250}$

$$x^2 + (y+1)^2 = 250$$

8) Center $(4, -14)$

Point on the Circle $(6, 11)$

$\Delta x = 2 \quad \Delta y = 25$

dist/r = $\sqrt{(2)^2 + (25)^2}$

$r = \sqrt{4 + 625} = \sqrt{629}$

$$(x-4)^2 + (y+14)^2 = 629$$

9) Prove or disprove that the points $A(8, 6)$, $B(8, -6)$ and $C(-10, 0)$ are the vertices of an isosceles triangle inscribed in the circle centered at the origin Q and passing through the point $P(-3, \sqrt{91})$.

dist/r b/ origin & pt A = $\sqrt{(8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$

dist/r b/ origin & pt P = $\sqrt{(-3)^2 + (\sqrt{91})^2} = \sqrt{9 + 91} = \sqrt{100} = 10$

B & C appear to work as well
P & P lies on the circle Q.

10) A new resort is being built on the shore of a lake that is roughly circular. Claire also lives on the lakeshore and she finds that the new resort is directly across the lake from her house. If the lake is put on a coordinate plane with x and y in miles, the coordinates of Claire's house are $(0.5, -1.2)$ and the coordinates of the new resort are $(-0.5, 1.2)$.

a) Is the center of the lake at the origin? Explain. yes see below

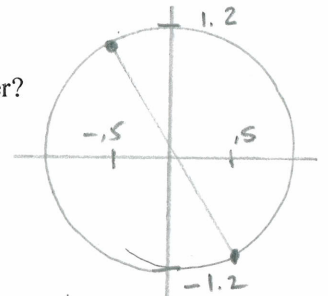
b) Find an equation that models the shoreline of the lake. $x^2 + y^2 = 1.69$

c) If Claire's boat is sitting at the coordinates of $(0.3, 1.25)$, is her boat in or out of the water?

a) Midpt of Houses has to be origin $(\frac{0.5 + (-0.5)}{2}, \frac{-1.2 + 1.2}{2}) = (\frac{0}{2}, \frac{0}{2}) = (0, 0)$

b) $r = \sqrt{(0.5)^2 + (-1.2)^2} = 1.3 \quad r^2 = (1.3)^2 = 1.69$

c) Dist Claire's boat from middle of lake (origin) = $\sqrt{(0.3)^2 + (1.25)^2} = 1.29$ Just barely in lake



11) A local television station in Marshall County has a range of 50 miles.

a) Write an equation that represents the region covered by this television station.

b) Can a person who lives 18 miles to the East and 35 miles North of the station watch this TV station?

a) $x^2 + y^2 = 2500$

b) dist = $\sqrt{(18)^2 + (35)^2} = 39.36 \text{ miles} < 50 \text{ miles, so yes they can watch TV.}$

- 12) Melissa lives at the corner of 3rd Street and 28th Avenue. Her sister Rebecca lives at the corner of 27th Street and 16th Avenue. Find the cross street that:

N/S

a. is halfway between their homes

$$\text{mid pt} = \left(\frac{3+27}{2}, \frac{28+16}{2} \right) = (15, 22)$$

b. is $\frac{2}{3}$ of the way from Melissa's to Rebecca's

$$X = 3 + \frac{2}{3} \left(\frac{24}{1} \right) = 3 + 16 = 19 \quad Y = 28 + \frac{2}{3} \left(\frac{-12}{1} \right) = 28 - 8 = 20$$

c. separates their homes in a ratio of 3 : 1

$$X = 3 + \frac{3}{4} \left(\frac{24}{1} \right) = 3 + 18 = 21 \quad Y = 28 + \frac{3}{4} \left(\frac{-12}{1} \right) = 28 - 9 = 19$$

d. separates their homes in a ratio of 1 : 5

$$X = 3 + \frac{1}{6} \left(\frac{24}{1} \right) = 3 + 4 = 7 \quad Y = 28 + \frac{1}{6} \left(\frac{-12}{1} \right) = 28 - 2 = 26$$

- (19, 20)
- (21, 19)
- (7, 26)

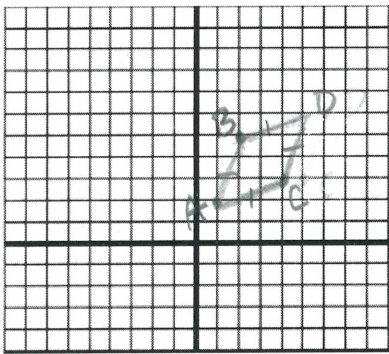
- 13) ABCD has vertices A (1, 2) B (2, 5) C (4, 3) D (5, 6). Determine which type of quadrilateral this is. Prove your assertion.

$$\overline{BD} = \sqrt{(3)^2 + (1)^2} = \sqrt{10} \quad \overline{AC} = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

$$\overline{AB} = \sqrt{(1)^2 + (3)^2} = \sqrt{10} \quad \overline{CD} = \sqrt{(1)^2 + (3)^2} = \sqrt{10}$$

Slopes of \overline{BD} & $\overline{AC} = 1/4$
 Slopes of \overline{AB} & $\overline{CD} = 3/1$

ABDC is a rhombus



- 14) EFGH has vertices E (4, 1) F (-2, 3) G (2, -5) H (-4, -3). Determine which type of quadrilateral this is. Prove your assertion.

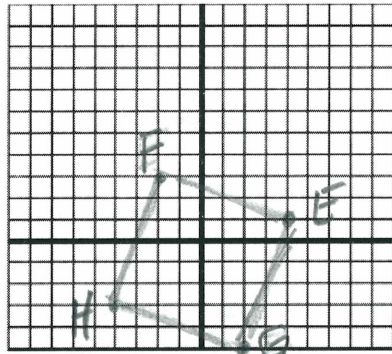
$$\overline{FE} = \sqrt{(6)^2 + (-2)^2} = \sqrt{40} = 2\sqrt{10}$$

$$\overline{EH} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$$

Slope of $\overline{HE} = \overline{FE} = 1/3$
 Slope of $\overline{FH} = \overline{EG} = 3$

Slopes are neg reciprocals, so lines are $\perp 90^\circ$.

FEGH is a SQUARE



- 15) Determine if point A lies on a circle with center C and point P on the circle.

a) A(5, 0) C(0, 0) P(3, 4)

$$CP = r = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$CA = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$$

Yes, P & A lies on circle C.

b) A(0, 4) C(2, -1) P(5, 3)

$$CP = r = \sqrt{(3)^2 + (2)^2} = \sqrt{13}$$

$$CA = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

Yes, P & A lies on circle C.

- 16) Write the equation of the lines below in slope-intercept form.

a. Through (-4, 5) and parallel to $y = -\frac{3}{2}x - 5$

$$5 = -\frac{3}{2}(-4) + b \quad y = -\frac{3}{2}x - 1$$

$$5 = 6 + b$$

$$b = -1$$

b. Through (4, 1), perpendicular to $y = -2x - 2$

$$m = \frac{1}{2}$$

$$1 = \frac{1}{2}(4) + b$$

$$1 = 2 + b$$

$$b = -1$$

$$y = \frac{1}{2}x - 1$$